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Theoretical Modeling of Rapid Surface Vaporization with Back Pressure

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In this paper, a theoretical model is developed for rapid surface vaporization into a surrounding ambient atmosphere. The primary emphasis is on metallic surfaces. Power input levels are taken to be low enough so that thermodynamic equilibrium can be assumed before phase change to gas. At high evaporation rates there will be a narrow Knudsen layer region in the gaseous flow just outside the phase interface in which translational nonequilibrium prevails. The modeling treats this layer as a gasdynamic discontinuity and approximate jump conditions are derived. The flow Mach number just outside the Knudsen layer is dictated by the state well away from the surface. Its determination is discussed both for a simple model of transient flow induced by a laser pulse and for the general case of time-varying power input. Examples assume the metallic surface is aluminum and the surrounding air pressure ranges from 1 atm to hard vacuum.

I. Introduction

THE genesis of this study was a need to understand the interaction of a high-power pulsed laser beam with a solid surface. The basic model can also be applied to cw laser interaction. Under sufficient energy loading all surfaces will ablate, and the simplest example of ablation is evaporation. Attention here is restricted to evaporation of a single compound with one well-defined boiling point. The primary emphasis is on metallic surfaces. The vaporization process typically involves a phase change from solid to liquid and then liquid to gas, but direct solid to gas phase change is possible. As a convenience, only liquid to gas phase change will be referred to in what follows.

The theoretical model developed applies when convective dominates diffusive evaporation. It is also assumed that thermodynamic equilibrium exists in the liquid. This assumption is reasonable for applications within the atmosphere where propagation requirements through aerosols[1] restrict practical laser intensities to less than 10^7 W/cm^2 or so for pulsed lasers. It would have to be re-examined for space applications.

When the vapor pressure is equal to the ambient pressure (saturated condition), the gas-phase distribution function consists of a half-range Maxwellian with zero mean velocity for forward evaporated particles and the same for back-scattered particles. The composite is then a Maxwellian distribution function and the gas is in translational equilibrium. As the vapor pressure becomes larger than ambient, the distribution function for forward evaporated particles remains a half-range Maxwellian if the liquid is near thermodynamic equilibrium, but in the limit of strong evaporation there are relatively few back-scattered particles at the phase interface. In other words, the gas near the phase interface is not in translational equilibrium when the evaporation rate is large—as will be the case at laser intensities of interest. Translational equilibrium is achieved within a few mean free paths by collisions between particles in a Knudsen layer region. This process has previously been modeled for evaporation into vacuum.[2] Here, the situation envisaged has the surface surrounded by air at various pressures.

The theoretical model of the Knudsen layer employed is based on treating that region as a gasdynamic discontinuity across which certain jump conditions expressing conservation mass, momentum, and energy are to be applied. This approach was originally employed by Anisimov[3] and is valid when the source Reynolds number of the flow is large.[2] One important difference between this analysis and that of Anisimov is that he considers only the strong vaporization limit, wherein the flow just outside the Knudsen layer is sonic. The flow Mach number is actually dictated by the flow state outside the Knudsen layer and is only sonic when the vapor pressure is large compared to the ambient pressure. Determination of the Mach number is discussed both for a simple model of the transient flow induced by a laser pulse and for the general case. The treatment of the Knudsen layer will handle perfect gases with internal degrees of freedom. However, examples are restricted to a monatomic vapor.

II. Knudsen Layer Jump Conditions

Let f be the gas-phase distribution function and ξ,η,ζ be (Cartesian) particle velocity components. The following quantities should be preserved across a very thin Knudsen layer if the vapor is a monatomic gas:

$$\int \xi f \mathrm{d}V = \text{mass flux}$$

$$\int \xi^2 f \mathrm{d}V = \text{momentum flux}$$

$$\int \tfrac{1}{2}(\xi^2+\eta^2+\zeta^2)\,\xi f \mathrm{d}V = \text{translational energy flux} \qquad (1)$$

where $\mathrm{d}V = \mathrm{d}\xi\mathrm{d}\eta\mathrm{d}\zeta$ is the volume element in velocity space and ξ is the velocity component normal to the vaporizing surface. The flow is treated as locally one-dimensional as is usual for a gasdynamic discontinuity. Evaluation of the integrals in Eq. (1) proceeds by what is essentially a Mott-Smith approach for treating rapid flow state changes.[4]

Translational equilibrium should be achieved at the outer edge of the Knudsen layer, so that a Maxwellian distribution function is assumed there.

$$f_2 = \rho(2\pi RT)^{-3/2}\exp\left[-\frac{(\xi-u)^2+\eta^2+\zeta^2}{2RT}\right] \qquad (2)$$

where $R=k/m$ is the gas constant and ρ,u, and T are the local gas density, mean velocity, and temperature, respectively. At

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the phase interface, the distribution function is taken to be

$$f_1 = \begin{cases} \rho_s (2\pi RT_s)^{-3/2} \exp\left[-\dfrac{\xi^2+\eta^2+\zeta^2}{2RT_s} \right] & \xi > 0 \\ \beta f_2 & \xi < 0 \end{cases} \tag{3}$$

The term for $\xi > 0$ assumes emitted particles were in equilibrium with the liquid before vaporization, so that ρ_s is properly interpreted as the saturated vapor density at the liquid temperature T_s. The second term accounts for back-scattered particles and is taken to be proportional f_2 lacking better information.

For a non-monatomic gas, energy conservation is expressed by augmenting the translational energy flux by terms involving the energy per unit mass due to internal degress of freedom, e_{int}, which is passively transported by an appropriate mass flux. With the assumed distribution functions, the additional terms are $e_{\text{int}}(T) \int \xi f_2 \, dV$ at the outside edge of the Knudsen layer and

$$e_{\text{int}}(T_s) \int \xi f_1 \, dV_+ + e_{\text{int}}(T) \int \xi f_1 \, dV_-$$

at the phase interface, where $dV_\pm$ are volume elements in the half-spaces $\xi \gtrless 0$. For a perfect gas, the internal energy per unit mass is $e = RT/(\gamma - 1)$ with γ being the (constant) ratio of specific heats. The portion of that energy associated with translational degrees of freedom is $3RT/2$. Therefore, the energy associated with internal degrees of freedom is:

$$e_{\text{int}}(T) = (5 - 3\gamma) RT/2(\gamma - 1) \tag{4}$$

Note that this expression vanishes for a monatomic gas ($\gamma = 5/3$).

Various assumptions can be made about what happens when a back-scattered particle arrives at the phase interface. Here, it will be assumed that all back-scattered particles condense. According to Williams,[5] this is typically correct for metals with monatomic vapor. Conservation of mass, momentum, and energy across the Knudsen layer is then modeled by the following jump conditions:

$$\rho u = \rho_s \sqrt{\frac{RT_s}{2\pi}} + \beta\rho \sqrt{\frac{RT}{2\pi}} \left[\sqrt{\pi} m \operatorname{erfc}(m) - e^{-m^2} \right]$$

$$\rho(u^2 + RT) = \frac{1}{2} \rho_s RT_s + \beta\rho RT \left[\left(m^2 + \frac{1}{2} \right) \operatorname{erfc}(m) - \frac{m}{\sqrt{\pi}} e^{-m^2} \right]$$

$$\rho u \left(\frac{5}{2} RT + \frac{1}{2} u^2 \right) = \rho_s \sqrt{\frac{RT_s}{2\pi}} \left[2RT_s + \frac{5 - 3\gamma}{2(\gamma - 1)} R(T_s - T) \right] + \beta\rho RT \sqrt{\frac{RT}{2\pi}} \left[m \left(m^2 + \frac{5}{2} \right) \sqrt{\pi} \operatorname{erfc}(m) - (m^2 + 2) e^{-m^2} \right] \tag{5}$$

where $m = u/\sqrt{2RT}$ and $\operatorname{erfc}(m) = (2/\sqrt{\pi}) \int_m^\infty e^{-v^2} dv$ is the complementary error function. With a bit of manipulation, these become

$$\frac{T}{T_s} = \left[\sqrt{1 + \pi \left(\frac{\gamma - 1}{\gamma + 1} \frac{m}{2} \right)^2} - \sqrt{\pi} \frac{\gamma - 1}{\gamma + 1} \frac{m}{2} \right]^2$$

$$\frac{\rho}{\rho_s} = \sqrt{\frac{T_s}{T}} \left[\left(m^2 + \frac{1}{2} \right) e^{m^2} \operatorname{erfc}(m) - \frac{m}{\sqrt{\pi}} \right] + \frac{1}{2} \frac{T_s}{T} \left[1 - \sqrt{\pi} m e^{m^2} \operatorname{erfc}(m) \right]$$

$$\beta = \left[(2m^2 + 1) - m \sqrt{\frac{\pi T_s}{T}} \right] e^{m^2} \frac{\rho_s}{\rho} \sqrt{\frac{T_s}{T}} \tag{6}$$

The Knudsen layer analysis provides no information about the value of m. Since the flow Mach number just outside the Knudsen layer $M = u/\sqrt{\gamma RT} = m\sqrt{2/\gamma}$, this is equivalent to saying that M can be freely specified without violating conservation of mass, momentum, and energy across the Knudsen layer. This result is not unusual because the same thing is true of the Rankine-Hugoniot jump conditions for a shock wave. The value of M will be determined by the flow state away from the Knudsen layer and the requirement that the Knudsen layer remains attached to the phase interface. A more detailed anlaysis[2] shows that translational equilibrium prevails at large-source Reynolds number if the flow is supersonic. This implies that application of the jump conditions should be restricted to $M \leq 1$.

For now, M will be treated as a parameter between zero and one. Note that the complementary error function is well-approximated by[6]:

$$e^{m^2} \operatorname{erfc}(m) \approx 0.34802t - 0.09588t^2 + 0.74786t^3 \tag{7}$$

with

$$t = 1/(1 + 0.47047m)$$

The absolute error is less than 10^{-4}. Table 1 results from Eqs. (6) and (7) for $\gamma = 5/3$. The pressure $p = \rho RT$ and $T_0 = (1 + (\gamma - 1)M^2/2)T$ is the stagnation temperature of the gas leaving the Knudsen layer. The last column gives the fraction of the emitted mass flux which actually leaves the Knudsen layer. Note that the mass fraction would not be zero for $M = 0$ if diffusive evaporation were accounted for. Convection should dominate diffusion for $M \gtrsim 0.05$.

III. Idealized Flow Induced by Surface Vaporization

An idealization of the flow induced by a laser pulse results if it is assumed that: 1) the laser intensity is a top-hat profile; 2) vaporization starts instantaneously at the beginning of the laser pulse ($t = 0$); 3) the mass efflux from the surface is steady during the pulse; 4) the surrounding air is uniform and quiescent; and 5) the surface recession rate is very slow compared to the gas efflux velocity. The assumptions about

Table 1 Flow property ratios across the Knudsen layer for $\gamma = 5/3$

M	ρ/ρ_s	T/T_s	p/p_s	β	T_0/T_s	$\rho u/\rho_s\sqrt{RT_s/2\pi}$
0	1	1	1	1	1	0
0.05	0.927	0.980	0.908	1.007	0.981	0.148
0.1	0.861	0.960	0.827	1.017	0.964	0.273
0.2	0.748	0.922	0.690	1.051	0.935	0.465
0.4	0.576	0.851	0.490	1.215	0.896	0.688
0.6	0.457	0.785	0.358	1.682	0.879	0.786
0.8	0.371	0.725	0.269	2.947	0.879	0.817
1.0	0.308	0.669	0.206	6.287	0.892	0.816

the vaporization process may be reasonable for a surface covered by metallic flakes, fibers, etc., which are in poor thermal contact with the bulk material at times before the backing surface heats up. The steady-state surface recession rate is $\rho_g u_g/\rho_\ell$ by mass conservation, where subscripts g and ℓ refer to gas and liquid. Therefore, the last assumption is valid if $\rho_\ell \gg \rho_g$, as is typically true.

Consider first the case in which the flow just outside the Knudsen layer is subsonic. The flow then has the structure depicted in Fig. 1. The flow discontinuities move at constant speed in the idealized flow, and flow properties are uniform between them. Vapor is in region ③ and air in region ① and ②. The Knudsen layer is taken to be stationary at $x=0$, which is correct in a reference frame attached to the phase interface and moving at its constant recession rate. There is no need to correct the gas flow speed to account for the moving reference frame, as long as the recession rate is comparatively slow as assumed.

With stationary air in region ①, Rankine-Hugoniot relations[7] give the velocity behind the shock wave as

$$u_2 = a_1\left(\frac{p_2}{p_1}-1\right)\Big/\gamma_1\sqrt{1+\frac{\gamma_1+1}{2\gamma_1}\left(\frac{p_2}{p_1}-1\right)} \tag{8}$$

where $a_1=\sqrt{\gamma_1 R_1 T_1}$, $\gamma_1=1.4$ is the ratio of specific heats for air, $R_1=\mathcal{R}/\mathfrak{M}_1$, $\mathcal{R}=8.314$ J/gmol K is the universal gas constant, and $\mathfrak{M}_1=29$ g/gmol is the molecular weight of air. Across the contact discontinuity, $u_3=u_2$ and $p_3=p_2$. Finally, from Eq. (6), the jump conditions across the Knudsen layer are:

$$\frac{T_3}{T_s}=\left(\sqrt{1+\pi\left(\frac{\gamma_3-1}{\gamma_3+1}\frac{m}{2}\right)^2}-\sqrt{\pi}\frac{\gamma_3-1}{\gamma_3+1}\frac{m}{2}\right)^2$$

$$\frac{\rho_3}{\rho_s}=\sqrt{\frac{T_s}{T_3}}\left[\left(m^2+\frac{1}{2}\right)e^{m^2}\operatorname{erfc}(m)-\frac{m}{\sqrt{\pi}}\right]$$

$$+\frac{1}{2}\frac{T_s}{T_3}\left[1-\sqrt{\pi}me^{m^2}\operatorname{erfc}(m)\right] \tag{9}$$

where $m=u_3/\sqrt{2R_3T_3}$. The metal will be taken to be pure aluminum, in which case $\gamma_3=5/3$ is the ratio of specific heats of the vapor, $R_3=\mathcal{R}/\mathfrak{M}_3$, and $\mathfrak{M}_3=27$ g/gmol is the molecular weight of the vapor.[3]

Define $M_3=u_3/\sqrt{\gamma_3 R_3 T_3}$ as the flow Mach number behind the contact discontinuity and note that M_3 is related to the pressure ratio across the shock wave by Eq. (8) and the fact that $u_2=u_3$. The inverse relationship results by solving a quadratic equation for p_2/p_1. The solution of physical interest is:

$$\frac{p_2}{p_1}=1+\gamma_1 M_3\frac{a_3}{a_1}\left[\frac{\gamma_1+1}{4}M_3\frac{a_3}{a_1}+\sqrt{1+\left(\frac{\gamma_1+1}{4}M_3\frac{a_3}{a_1}\right)^2}\right] \tag{10}$$

Now,

$$p_3=p_2,\ \frac{p_s}{p_1}=\frac{p_3}{p_1}\Big/\frac{p_3}{p_s}\ \text{and}\ \frac{p_3}{p_s}=\frac{\rho_3}{\rho_s}\frac{T_3}{T_s}$$

Therefore, the relations (9) and (10) allow explicit evaluation of p_s/p_1 as a function of M_3 and T_s/T_1. This leads to the solid curves shown in Fig. 2 for $0<M_3\le 1$. There can be no flow for $p_s\le p_1$, because the vaporization model excludes diffusive evaporation. The meaning of the dashed curves will be discussed later.

Nonequilibrium processes within the Knudsen layer cannot drive the flow to greater than sonic velocity within that zone. If the saturated vapor pressure is high enough, there is a

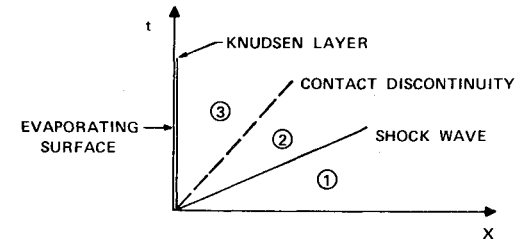

Fig. 1 Subsonic flow structure.

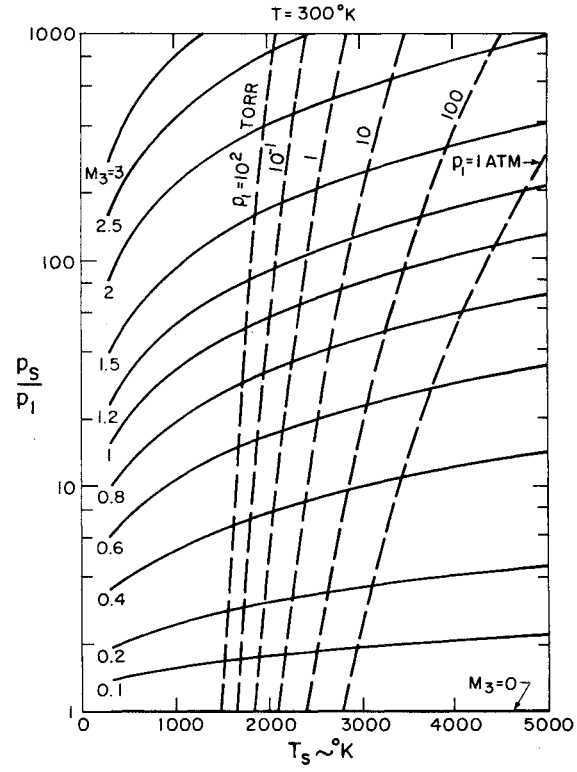

Fig. 2 Allowed flow states for aluminum vapor.

centered rarefaction fan in the flowfield which takes the sonic flow outside the Knudsen layer to supersonic speed. The flow structure in this case is as shown in Fig. 3. Conditions at the outside edge of the Knudsen layer will be denoted by subscript 4 and $M_4=1$ in what follows.

The Riemann invariant $P=2a/(\gamma_3-1)+u$ is constant across the Q rarefaction fan.[7] The constant can be evaluated by noting that $u_4=a_4$. Since the flow is also isentropic through the rarefaction fan, the sound speed and pressure ratios across it are given in terms of M_3 by:

$$\frac{a_4}{a_3}=\left(\frac{p_4}{p_3}\right)^{(\gamma_3-1)/2\gamma_3}=\frac{2}{\gamma_3+1}+\frac{\gamma_3-1}{\gamma_3+1}M_3 \tag{11}$$

The relationship between M_3 and p_2/p_1 given in Eq. (10) remains valid. Since the flow outside the Knudsen layer is choked, Table 1 gives $p_4/p_s=0.206$ and $T_4/T_s=0.669$ for $M_4=1$. Now,

$$p_2=p_3,\ \frac{p_s}{p_1}=\frac{p_4}{p_3}\frac{p_3}{p_1}\Big/\frac{p_4}{p_s}$$

and

$$\frac{a_3}{a_1}=\sqrt{\frac{\gamma_3\mathfrak{M}_1}{\gamma_1\mathfrak{M}_3}\frac{T_s}{T_1}\frac{T_4}{T_s}}\Big/\frac{a_4}{a_3}$$

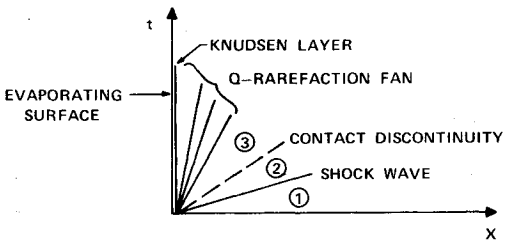

Fig. 3 Supersonic flow structure.

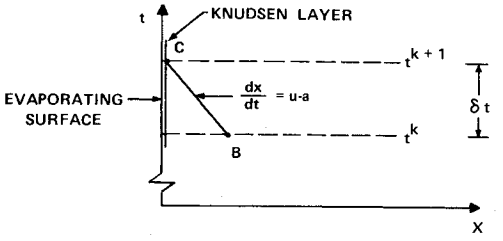

Fig. 4 Characteristic intersection problem.

Again, explicit relationships are available to evaluate p_s/p_l as a function of M_3 and T_s/T_l. This leads to the solid curves shown in Fig. 2 for $M_3 > 1$. These curves and those for $M_3 \leq 1$ define the locus of states that are permissible based on fluid mechanic considerations alone.

A unique flow state is expected physically. Such uniqueness follows by noting that the saturated vapor pressure is known as a function of the liquid temperature T_s. Here a Clasius-Clayperon relationship will be used:

$$\frac{p_s}{p_l} = \exp\left[\frac{\mathfrak{M}_3 L_v}{\mathfrak{R} T_{v_l}}\left(1 - \frac{T_{v_l}}{T_s}\right)\right] \tag{12}$$

where L_v is the latent heat of vaporization and T_{vl} is the value of T_s required to produce a vapor pressure p_l. For aluminum $L_v = 10875$ J/g and $T_v = 2790$ K for $p_l = 1$ atm are reasonable choices.[8] Values of T_{vl} for other values of p_l (expressed in atmospheres) are obtained from

$$T_{v_l} = T_v \Big/ \left(1 - \frac{\mathfrak{R} T_v}{\mathfrak{M}_3 L_v} \ell n p_l\right) \tag{13}$$

The dashed curves in Fig. 2 result from Eq. (12) for representative values of p_l. The intersections of the two sets of curves in Fig. 2 define the flow states which will be observed according to the model employed.

The liquid temperature T_s is established by energy-balance considerations. It is most convenient to compute the required power as a function of T_s. The absorbed incident radiation q_i should be divided between conductive heat loss to the surroundings q_c and latent heat carried away by vapor leaving the Knudsen layer. Thus,

$$\dot{q}_i = \dot{q}_c + \rho_3 u_3 L_v \tag{14}$$

It follows that

$$(\dot{q}_i - \dot{q}_c)/L_v \rho_s \sqrt{\frac{R_3 T_s}{2\pi}} = \rho_3 u_3/\rho_s \sqrt{\frac{R_3 T_s}{2\pi}} \equiv \phi \tag{15}$$

where $\phi = 2\sqrt{\pi} m (p_3/p_s)/\sqrt{(T_3/T_s)}$ if the flow leaving the Knudsen layer is subsonic and $\phi = 0.816$ if it is sonic. Note that $\rho_s = p_s/R_3 T_s$ and that the vapor pressure is given by Eq. (12).

Values of T_s corresponding to each value of M_3 can be determined graphically from Fig. 2. Then Eqs. (12) and (15) can be applied to determine $\dot{q}_i - \dot{q}_c$. The results of this exercise for $p_l = 1$ atm and $M_3 \leq 1$ are given in Table 2. The numbers noted provide a lower bound on the absorbed radiation ($\dot{q}_i$), which is generally a small fraction of the incident intensity, I_i, at typical laser wavelengths.[9] The flux levels involved and the fact that $I_i \lesssim 10^7$ W/cm^2 to propagate through the atmosphere make it clear that the flow outside the Knudsen layer is typically going to be subsonic in low-altitude environments. This will not be true at high altitude. For example, to produce sonic flow for $p_l = 10$ Torr only requires $T_s \approx 2800$ K and $\dot{q}_i - \dot{q}_c \approx 3.9 \times 10^4$ W/cm^2.

The approximate physical modeling should be kept in mind in considering these results. The Knudsen layer jump conditions, Eq. (6), can be expected to be only qualitatively correct. Another point is the use of a Clasius-Clayperon relation with constant L_v and T_{vl}. This can only be correct for a restricted range on T_s. The correct vapor pressure curve for aluminum should have been used in constructing Fig. 2 and determining $\dot{q}_i - \dot{q}_c$ in Table 2. Vapor pressure data for aluminum with $p_s > 1$ atm were not found in the literature. Finally, for values of T_s sufficiently larger than 2800 K, the critical point can be approached for aluminum. Van der Waal's effects would then cause significant deviation from the perfect gas behavior assumed for the vapor in constructing the Knudsen layer jump conditions.

IV. Treatment for a General Transient Flow

Continue to consider one-dimensional flow for a moment, but now allow a general variation of incident laser flux, etc., with time. Since the Mach number just outside the Knudsen layer is a free parameter in the jump relations (6), there must be some way to determine it if a general transient flow is to be treated. This is provided by the requirement that the Knudsen layer remain attached to the phase interface. To illustrate the procedure, it will be adequate to assume that the phase interface is stationary in an appropriate moving reference frame and that terms due to acceleration of the reference frame can be neglected in the equations of motion of the gas.

Information about the flow state away from the Knudsen layer is carried by the incoming Mach line (e.g., $dx/dt = u - a$) if the flow is subsonic at the edge of the Knudsen layer. This leads to consideration of the characteristic intersection problem illustrated in Fig. 4. In the case of constant area flow with no friction or heat transfer, two characteristic relations apply along the Mach line CB[10]:

$$(p_C - p_B) = \langle \rho a \rangle (u_C - u_B)$$
$$(x_C - x_B) = \langle u - a \rangle \delta t \tag{16}$$

where $\langle \cdot \rangle$ denotes average value along CB and $x_C = 0$. All flow properties are taken to be known at time t^k, so values of p_B, etc., can be determined by interpolation for given x_B. Therefore, Eq. (16) together with the jump relations, vapor pressure as a function of T_s, and a surface energy balance relation(s) define a closed algebraic system which can be

Table 2 Idealized flow produced by laser interaction with aluminum

M_3	0	0.1	0.2	0.4	0.6	0.8	1.0
T_s ~ K	2790	2950	3100	3440	3760	4110	4480
$\dot{q}_i - \dot{q}_c \leq$ W/cm^2	0	2.65×10^4	8.24×10^4	3.96×10^5	1.13×10^6	2.74×10^6	5.8×10^6

solved iteratively to determine what is presumably a unique state at the outer edge of the Knudsen layer. Explicit time-differencing schemes probably can also be formulated.

The characteristics update procedure cannot be applied if $a_C \geq u_C$. In that case, the characteristic relations (16) are deleted and replaced by the requirement that the flow be sonic at the outside edge of the Knudsen layer: $u_C = a_C$. As mentioned before, nonequilibrium processes within the Knudsen layer cannot drive the flow to supersonic speed within that region.

The basic procedure just outlined can be applied to a general transient two- or even three-dimensional flowfield by employing a reference-plane characteristics treatment. That is, terms involving derivatives transverse to the phase interface are considered to be forcing terms in the equations of motion, which can be evaluated by straightforward finite differencing. Characteristic relations are derived in terms of time and the remaining spatial coordinate, treating the forcing terms as known. This leads to a generalized form of the relations (16), which when coupled with other relations noted above define the flow state just outside the Knudsen layer.

V. Conclusion

The modeling discussed in this paper provides a general, though approximate, treatment of surface vaporization into a surrounding environment at pressures ranging from ambient sea level conditions to hard vacuum. A brief literature review has shown that no comparable treatment is currently available. It has important application to understanding the flow induced by a pulsed laser interacting with a surface. Some generalization would be required to treat materials which ablate with chemical reactions or for evaporating materials involving several compounds with disparate boiling points. The following important conclusions have been reached.

1) In the case of rapid surface vaporization, there is a thin zone in the gaseous region neighboring the phase interface which can be treated as a gasdynamic flow discontinuity.

2) Approximate jump conditions across this Knudsen layer region can be derived using kinetic theory and reasonable assumed forms for the gas-phase distribution function.

3) There is one free parameter in the jump relations and this can be determined by considering the flow state outside the Knudsen layer and requiring the Knudsen layer to remain attached to the phase interface.

4) Laser interaction with an aluminum surface at low altitudes in the Earth's atmosphere will typically involve subsonic flow outside the Knudsen layer.

5) At high altitudes or in hard vacuum, laser interaction with an aluminum surface will typically involve sonic flow at the edge of the Knudsen layer and supersonic flow outside of it.

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